

## COMPUTER-ENRICHED INSTRUCTION OF STATISTICAL METHODS

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### ABSTRACT

Any course in statistical methodology is hampered by substantial diversity of motivation, background and analytic maturity among the students within the class. This proposal suggests a way of using a computer to reduce the effects of this heterogeneity by having it produce instructional material tailored to the background of each student. Previous activities in this direction are described and serve as a basis for suggesting two alternative projects, either of which could have substantial long term effects on Statistics instruction in the United States. The first project deals with the creation of a computer software package which can produce homework problems involving randomly generated data and can separately produce instructive feedback including the associated solutions with comments about typical errors. The tailoring will be accomplished by embedding the randomly generated data in a subject-matter setting of direct interest to the student. The second and larger project expands the preceding project by providing for development of its capability for allowing student-computer interaction about the students' solutions as soon as they are completed. The first project will progress toward its objectives in a well defined and rather certain manner. The proposed implementation for the second project is somewhat less specific than that for the first project because its first stage includes planning for the subsequent two stages. Since the larger project would have the greatest long term impact on Statistics instruction, funding is sought for it as a matter of first priority. If funding for the larger project is not possible at this time, then funding is sought for the first and smaller project. But it should be recognized that the smaller project has much more limited goals and it would be pursued differently by itself than if it were a part of the larger project.

### THE PROBLEM

The subject of Statistics can be taught in several ways. It can be taught as a course in Mathematics, that is, as mathematical or theoretical Statistics. Or it can be taught from a purely procedural viewpoint by considering only the arithmetic associated with statistical inference; students with a limited mathematical background are commonly introduced to Statistics this way. Such an approach leaves a lot to be desired even though it is widely used for teaching statistical methodology. Any user of statistical techniques, regardless of his

background, must understand much more than the arithmetic of Statistics. He must be able to

1. recognize important features of an experiment;
2. translate a subject-matter concern into a statistical one;
3. select an appropriate statistical procedure;
4. apply the selected procedure correctly;
5. translate the statistical conclusion back into the experimental realm.

Since a course in statistical methodology should teach the budding researcher how to apply statistical techniques to his experiments, it must speak to each of these issues regardless of the mathematical maturity or background.

A typical class beginning a statistical methods course is heterogeneous not only with regard to mathematical maturity, but also relative to motivation, previous insight into Statistics and disciplinary interests. Now consider teaching a class of 150-200 students; a class which is extremely variable in these and other ways. A rational sectioning scheme, based upon prerequisites and disciplinary interests, produces at least 15 sections. Such sectioning is not desirable because:

1. few schools have the available and qualified manpower for teaching this number of sections;
2. an instructor with only one large class has more time to prepare supporting instructional materials than if he is teaching the same number of students split into several sections;
3. much learning takes place through interaction between students; some heterogeneity enhances this process;
4. all of the students should become familiar with the same concepts and general techniques, regardless of their disciplinary interest.

Thus, in the College of Agriculture at Cornell University, we have decided to teach one course in statistical methodology for advanced undergraduates and graduate students while seeking ways to accommodate the heterogeneity, or when possible, to capitalize upon it. Our present strategy is to present concepts and a few well chosen examples in lecture, to provide students with an opportunity to interact with graduate teaching assistants in weekly, discipline-oriented small group discussions, and to have the students apply the concepts and techniques to weekly laboratory exercises.

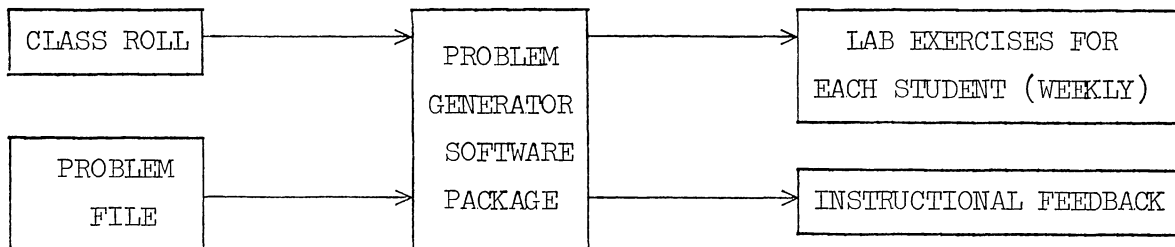
This proposal is specifically directed toward improving the laboratory experience through use of the computer, but it should be viewed within the broader framework outlined above. Instructional materials are simultaneously being developed in conjunction with the other parts of this approach.

THE OBJECT

How might the computer be used to

1. reduce motivational hangups of nonanalytically-oriented students?
2. make the instructional material as relevant to students' disciplinary concerns as possible?
3. provide good instructional feedback on answers to their laboratory exercises?
4. adapt the teaching-learning process in any reasonable manner to improve understanding of concepts?

Consider a computer-based system of the following type:



Depending upon coding with each student's name, the software package (or for brevity, the system) should be capable of producing different, but equivalent, problems which relate directly to the student's disciplinary concerns. Each problem should consist of a verbal description, data randomly generated according to a probability model specified by the instructor, and questions which the student is to answer. The fact that each student would receive a set of laboratory material labeled with his name (a trivial, but apparently significant thing), containing his own data, and framed in a setting which is of direct interest to him, should go a long way toward reducing the motivational hangups and increasing the apparent relevance of the topics under consideration.

The random generation of data opens a whole new way of illustrating concepts and of encouraging interactive learning. Statistics deals with probabilistic approaches to data-analysis problems. When the laboratory problems have been chosen with sufficient foresight, a probabilistic idea can be illustrated by using the whole class as a simulation mechanism. For example, power (probability of rejecting a false hypothesis) is influenced by sample size ( $n$ ) and the size of deviation from the null hypothesis. Suppose each student receives several test-of-hypothesis problems which differ probabilistically only in  $n$ . Some of the randomly generated samples will lead to rejection of their respective hypotheses, others

will not. If all of the hypotheses and the respective generating models differ by the same amount, then more students should reject those hypotheses associated with larger values of  $n$ . When their results are summarized in class by a show of hands, they see that more and more people correctly reject the null hypothesis as  $n$  increases. This is a forceful way for them to learn about power and things which influence it. This approach is also applicable to other concepts. As far as interactive learning is concerned, students will always cooperate in solving problems of the type under discussion. But if each student has a different set of data, the cooperation must be at the level of "How should the problem be approached?" rather than at the level of "What is the numerical answer?". This will increase the understanding of Statistics.

A computer-based feedback process is an improvement over hand methods for several reasons. There is not time enough to write instructive comments on each student's lab exercise as it is graded. But this feedback could take place at several levels:

1. numerical answers corresponding to each student's data;
2. numerical answers plus the same comment to all students who have the same questions; or
3. comments which are conditional on what a student has done with his data.

This last possibility would obviously require student-computer interaction. Further, if the feedback process were on line, it could be initiated by the student as soon as he has completed his problems, much sooner than presently is possible with hand methods. The immediate feedback has obvious pedagogical value.

Such a system should allow the instructor substantial flexibility. For example, it should accommodate at least 15 kinds of problems; for each problem type several reasonable questions exist. An adequate problem file should contain several hundred problems because each student should receive 50 - 60 problems per term. If the disciplines were grouped into 10 sets (applied plant science, ecology, nutrition, etc.), then it would appear that 500 - 600 problems would be needed. Somewhat fewer problems (300 - 400) are actually needed because some problems are equally relevant to several disciplinary areas.

#### PAST AND PRESENT ACTIVITY

The writer has been experimenting with systems similar to the type described above since the summer of 1966. The original work was done on a CDC 1604 here at

Cornell; subsequent work has been done on the evolving Cornell System 360. There is now available what might be called "a preliminary draft" of a system which has all of the desirable properties previously suggested except the interactive feedback of the computer with the student. This system, written entirely in FORTRAN IV, consists of a control program and major subprograms for problem generation and answer computation. These subprograms accomplish their tasks through about 150 subroutines, sometimes nested three-deep. Presently the system allows for generation of twelve types of problems and about 60 different answer approaches.

Preliminary utilization of the system during Fall term 1968 indicated that the system, though well conceived, needs substantial attention from two directions. The feedback part of the system needs a substantial amount of debugging (from an instructional, not computing, viewpoint). Our original planning had underestimated both the magnitude and importance of producing comments devoid of syntactic and statistical vagueness. The bugs pose a minor problem, though, compared to the time it takes to set up a complete laboratory exercise which fully utilizes the capabilities of the system. One complete laboratory exercise requires at least 40 hours of preparation (20 problems, two hours each), half of which the professor must do because a lot of serious questions about what the problems are to help teach, and how, arise during their writing. A graduate student can do the remaining adaptation work. This investment of time sounds prohibitive until one realizes that once a problem has been set up, it can be used year after year and really, there is no reason it shouldn't be used by professors and students at other schools. (A copy of an abbreviated set of problems is presented in the Appendix.)

#### IMMEDIATE NEED: DEVELOPMENT AND OPTIMIZATION

Time is needed to develop problems to go through this system before its instructional potential can be realized. It will take a faculty member and a graduate student in Statistics several months to develop a good, complete set of exercises. For three years students in Statistical Methods I, II have each been required to produce two potential problems (situations from their discipline) which might be adapted to this system. This set of 600 problems will serve as the beginning point in the development of the problem file.

The existing system needs a lot of programming attention to change it from a preliminary draft into a software package which can be made available to other schools. It was originally designed to allow easy addition and modification, a feature which should be maintained.

### THE NEXT STEP

The feedback process is the primary difficulty with the present system. The current approach is for the system to generate a set of problems for a student and separately a set of instructive comments and numerical answers which correspond to his problem and data. The logistics of a large class require that all students hand in their worked problems at the same time and that their "answer sheet" be handed back with their paper after grading. Any comparison of the student's and computer's answers must be done manually. The instructional value of the exercises could be substantially increased if the students could find out about their solutions as soon as they were done. The necessary software should be developed for accommodating this through typewriter terminals. If the student has an incorrect solution, the computer should be prepared to help him track it down.

A number of serious students have made a valid request which could be adapted to this type of interactive system. The request is that they would like access to a larger set of problems (than their own laboratory exercises) on which to "practice". In particular they want to see problems in experimental contexts, propose an approach to each and then receive some sort of feedback about the appropriateness of their proposed solution. This is a natural extension to having the computer help a student examine his answer to assigned problems. In particular this could be accomplished by providing all students with some sort of limited access to the whole problem file.

The ideas of this and the preceding section do not represent programmed instruction. Neither do they require that the student learn how to do computer programming. Instead, the object is to get the computer to provide the student with a greater wealth of experiences than he could otherwise encounter. Nevertheless some of the ideas being developed at Systems Development Corporation, specifically in the context of PLANIT, could be used here, but PLANIT does not and will not have the capability of accommodating the type of interaction proposed here.

### AVAILABILITY OF RESULTS

Results of the work proposed here would be made available to Statistics instructors in the United States. The ideas set out above generated substantial interest when they have been discussed at the annual meetings of the Statistics profession. Two levels of interest emerge. Some statisticians are interested in

the whole system as described above. Several have requested information about and/or copies of the preliminary version. Others are interested in the problem file because of its diversity of types of problems and disciplinary settings. The results will be made available to accommodate both of these interests.

Since the ultimate goal is to make this material generally available, interaction with Statistics instructors should take place during the development phase. It is on this point that the FLARCO association should be stimulating. Interaction with Statistics instructors at the cooperating schools should help us find additional instructional bugs which we have missed prior to general release of the software package.

### TWO ALTERNATIVE PROJECTS

The preceding material suggests two different projects: (1) Develop and optimize the system for generating problems, a system of which a "preliminary draft" exists, without any regard for student-computer interaction; (2) Pursue (1) while simultaneously planning and designing an interactive system, thereafter implementing the interactive system. These two projects obviously differ in strategy, magnitude and support requirements. Separate budgets are subsequently presented for each of the projects.

Consider first the problem generation system. Substantial progress probably can be made toward completing this project in two summers and the intervening academic year if the principal investigator could spend half time on it during the summers and be assisted by two graduate students for the entire period. During the academic year the principal investigator must work with the class which provided the motivation for the project, but some of this activity will relate directly to the project, namely, testing it on this class. Further, he must direct the work of the graduate assistants supported by the project, but both of these activities are project related and should consume about 1/5 of his time. One of the graduate students would be responsible for statistical aspects of the system, the other for computing aspects.

The larger project, to concurrently produce computer-student interaction, would require somewhat larger and longer-term financial support. During the first year of the larger project the interactive system would be planned and designed in conjunction with the problem-generation efforts; these latter efforts would be executed essentially as outlined in the preceding paragraph, except that

due respect would have to be given to possible ramifications in the student-computer interaction efforts. A research associate with experience in the teaching of Statistics (a Ph. D. Statistician) and a systems programmer would have to investigate available software systems for flexibility from a statistical viewpoint and adaptability to our existing System 360. Presumably both of these requirements would require that the selected system undergo some adaptation. During this adaptation a second systems programmer will be needed. During the investigation-of-software-systems stage the research associate and the systems programmer will probably have to travel quite a bit, at least to Systems Development Corporation, Science Research Associates and MIT, thus that budgetary item. During the second year the systems adaptation should be nearly completed and the research associate would begin the instructional implementation. During the third year the interactive system should be ready for serious testing on students. If the testing produces satisfactory results, documentation and information dissemination will be conducted during the latter part of the third year. Throughout the entire period the principal investigator and the two graduate students would continue to be involved in development of supporting materials and actual classroom implementation.

Three alternative levels of funding emerge, listed here from most to least desirable from the writer's viewpoint:

1. Fund the larger project for three years.
2. Fund the larger project for the first year with possible renewal the second and third years.
3. Fund only the smaller project.

The smaller project is quite worthwhile, but compared to the larger project it would have a smaller impact on Statistics instruction. It is for this reason that it is listed as being least desirable. Further, it might be very inefficient to fund the smaller project with the view of funding the interactive part of the larger project at a later date. This is because the problem generation part of the software package might have to be structured somewhat differently as a part of the interactive system than by itself. Still, if funding is available only for the smaller project, the writer would be delighted to have an opportunity to work on it. If the larger project is funded, three-year funding would be substantially better, from a personnel point of view, than one-year funding. It is going to be a difficult enough job to find a Ph. D. statistician with teaching



experience who would work on this project as a research associate. (The writer is aware of one such person who might be interested.) It would be much easier to find someone under a three-year funding situation than under a one-year situation. But again if only one-year funding is available, the writer would gladly accept it and plan to apply for subsequent funding later.

APPENDIX TO COMPUTER-ENRICHED INSTRUCTION OF STATISTICAL METHODS

Examples of the type of problems discussed in the preceding text are set out here. Laboratory Exercise 9 for Statistics 510, which the students were to turn in November 19, 1968, serves as the specific example. Each student received three computer-generated problems. The first was to illustrate how the t-statistic could be used on samples from unpaired populations; the second dealt with samples of pairs and provided an opportunity to consider the central limit theorem; the third sought to engage students in the translation of an experimental question into a statistical question which could be answered two ways, depending upon the assumptions.

At least twenty problems should have been used for this exercise (9 areas  $\times$  3 problems each = 27, but some could be used for two areas); time allowed for the development of only five. The first problem was framed in three settings, only one setting was available for each of the other two, but each student had his own sample of randomly generated data.

Copies of three laboratory exercises and their associated answer sheets follow. Note that the first problem has a different setting for the three exercises, but that statistically equivalent questions are asked for each. Observe further that the difference in the data for the second (and third) problems in the three exercises is a consequence of random sampling since the settings are the same. Even a cursory examination of the answer sheets which follow each of the exercises will show that they are poorly related to their associated problems. It is in this domain where we have the most work to do.

AC EX  
PROPOSED SAMPLE 1

NOVEMBER 19, 1968

PROBLEM 1 10 POINTS

STUDENTS IN THE LABORATORY SECTION OF AN ELEMENTARY PHYSIOLOGY COURSE WERE ASKED TO STUDY THE EFFECT OF RATE-OF-METABOLISM ON THE AMOUNT OF LIVER REGENERATION IN LABORATORY WHITE RATS. THE STUDENTS SURGICALLY REMOVED THE LEFT LOBE FROM THE LIVER OF 30 RATS. THE RATS WERE RANDOMLY DIVIDED INTO TWO GROUPS; ONE GROUP RECEIVED A THYROID EXTRACT WHICH PRODUCED HYPERTHYROIDISM (LEADS TO HIGH METABOLISM), AND THE OTHER GROUP RECEIVED A COMPOUND WHICH PRODUCED HYPOTHYROIDISM (LEADS TO LOW METABOLISM). AFTER 15 DAYS OF TREATMENT THE RATS WERE SACRIFICED, AND THE AMOUNT OF REGENERATED LIVER WAS DETERMINED FOR EACH RAT. THE WEIGHTS (IN GRAMS) OF REGENERATED LIVER ARE RECORDED BELOW. (EXPERIENCE INDICATES THAT THE RESPONSE IS APPROXIMATELY NORMALLY DISTRIBUTED.)

HYPERTHYROID	1.41	1.81	1.55	1.79	1.66	1.90	1.92	1.51
	1.79	1.94						
HYPOTHYROID	1.53	1.63	1.69	1.68	2.16	1.93	1.46	1.55
	1.54	1.90	1.73					

A. THE AVERAGE AMOUNT OF REGENERATION FOR HYPERTHYROID RATS, REPORTED BY STUDENTS OVER THE PAST 10 YRS., IS 1.5 GRAMS. IS THE CURRENT DATA CONSISTENT WITH THIS LONG TERM AVERAGE? (USE HYPERTHYROID DATA ONLY; LET  $\alpha = .01$ )

B. CONSTRUCT A .95 CONFIDENCE INTERVAL FOR THE POPULATION MEAN REGENERATION IN HYPOTHYROID RATS. (USE HYPOTHYROID DATA ONLY)

C. CURRENT PHYSIOLOGICAL THEORY SUGGESTS THAT HYPERTHYROID RATS WILL REGENERATE THE GREATEST AMOUNT OF LIVER. DOES THE DATA SUBSTANTIATE THIS? (ASSUME EQUAL VARIANCES; LET  $\alpha = .05$ )

D. CONSTRUCT A .95 CONFIDENCE INTERVAL ABOUT THE COMMON VARIANCE FOR THE POPULATIONS (HYPO-HYPERTHYROID RATS).

AL EX  
PROPOSED SAMPLE 1

STATISTICS 510, T-TEST  
LAB.NO. 9

SEAT NO.  
CODE NO. 5

NOVEMBER 19, 1968

PROBLEM 2 7 POINTS

A NUTRITION LABORATORY IS INTERESTED IN COMPARING TWO METHODS FOR MEASURING THE AMOUNT OF AMINO ACID SYNTHESIS UNDER VARIOUS DIETS. HOWEVER, THE MEASUREMENTS ARE TECHNICIAN DEPENDENT, I.E., SEVERAL TECHNICIANS WOULD PRODUCE DIFFERENT MEAN METHODS FOR THE SAME SAMPLE. SINCE AN APPROPRIATE EXPERIMENTAL DESIGN AND ANALYSIS COULD BE EMPLOYED TO REMOVE THE EFFECT DUE TO TECHNICIANS, THE NUTRITION LABORATORY DECIDED TO LET EACH OF THEIR 12 TECHNICIANS ANALYZE 10 AMINO ACID SAMPLES UNDER EACH METHOD OF MEASUREMENT. THE MEAN MEASUREMENTS OF THE TEN SAMPLES (IN MILLILITERS) FOR THE 11 TECHNICIAN ARE GIVEN BELOW.

	TECHNICIAN							
	1	2	3	4	5	6	7	8
METHOD 1	1.99	2.50	1.02	2.06	1.52	2.17	2.07	2.58
METHOD 2	1.90	2.16	0.20	1.92	0.78	1.67	1.27	2.94
	9	10	11					
METHOD 1	2.70	2.16	2.05					
METHOD 2	2.61	2.08	1.58					

A. A MEAN OF TEN SAMPLES WAS USED AS A MEASUREMENT INSTEAD OF USING THE INDIVIDUAL OBSERVATIONS. WHAT EFFECT DOES THIS HAVE ON THE VARIANCE? WHAT DISTRIBUTION WOULD YOU EXPECT THE DATA TO FOLLOW? EXPLAIN.

B. PLACE A .95 CONFIDENCE INTERVAL AROUND THE TRUE MEAN DIFFERENCE FOR THE TWO METHODS OF MEASUREMENT.

C. CONSTRUCT A .95 CONFIDENCE INTERVAL FOR THE POPULATION VARIANCE OF THE DIFFERENCE BETWEEN THE TWO METHODS OF MEASUREMENT.

AL EX  
PROPOSED SAMPLE 1

STATISTICS 510, T-TEST  
LAB.NO. 9

SEAT NO.  
CODE NO. 5

NOVEMBER 19, 1968

PROBLEM 3 8 POINTS

TWENTY PLOTS OF GROUND WERE PLANTED TO CORN. A RANDOMLY SELECTED SET OF TEN PLOTS RECEIVED A NEW TYPE OF PHOSPHORUS FERTILIZER; THE REMAINDER RECEIVED NOTHING. DID THIS TREATMENT INCREASE THE YIELD?

TREATMENT 5.82 5.72 5.92 6.05 5.90 6.37 5.75 6.18 5.74 5.99

CONTROL 5.54 5.63 5.03 4.73 4.78 5.01 4.79 5.23 4.89 5.12

A. DESCRIBE THE POPULATION STRUCTURE AND FRAME THE EXPERIMENTAL QUESTION IN A STATISTICAL MANNER.

B. ANSWER THE EXPERIMENTAL QUESTION WITHOUT ASSUMING NORMALITY OF YIELD DISTRIBUTION.

C. ANSWER THE EXPERIMENTAL QUESTION ASSUMING NORMALITY OF YIELD DISTRIBUTION AND EQUALITY OF VARIANCES.

## PROBLEM 1

10 POINTS

A

CALCULATED  $T = 3.88$  CRITICAL  $T = 3.25$  D.F. = 9  
 REJECT THAT MEAN = 1.5000  
 IN FAVOR OF MEAN NOT EQUAL TO 1.5000

B

ESTIMATED MEAN = 1.71  
 CONFIDENCE INTERVAL IS  $P(1.57 \leq \text{MEAN} \leq 1.85) = .95$

C

$H_0$ : HYPERTHYROID MEAN  $\leq$  HYPOTHYROID MEAN  
 $H_A$ : HYPERTHYROID MEAN  $>$  HYPOTHYROID MEAN  
 CALCULATED  $T = 0.22$  CRITICAL  $T = 1.73$  D.F. = 19  
 FAIL TO REJECT THAT MEAN OF X = MEAN OF Y

D

ESTIMATED COMMON VARIANCE = 0.04  
 CONFIDENCE INTERVAL IS  $P(0.02 \leq \text{VARIANCE} \leq 0.08) = .95$

## PROBLEM 2

7 POINTS

A

NO NUMERICAL ANSWER  
 THE CENTRAL LIMIT THEOREM ASSURES APPROXIMATE NORMALITY. THE VARIANCE OF SUCH A MEAN IS LESS THAN THAT OF AN INDIVIDUAL OBSERVATION.

B

ESTIMATED MEAN = 0.34  
 CONFIDENCE INTERVAL IS  $P(0.09 \leq \text{MEAN} \leq 0.59) = .95$

C

ESTIMATED VARIANCE = 0.14  
 CONFIDENCE INTERVAL IS  $P(0.07 \leq \text{VARIANCE} \leq 0.42) = .95$

## PROBLEM 3

8 POINTS

A

NO NUMERICAL ANSWER  
 THERE IS A CONCEPTUAL POPULATION OF CORN YIELDS WHEN THE FERTILIZER IS APPLIED; ANOTHER WHEN IT IS NOT. WE HAVE UNRESTRICTED RANDOM SAMPLES FROM EACH OF THESE. ARE THEIR DISTR. THE SAME?

B

$H_0$ :  $F_T(Y) \geq F_C(Y)$   
 $H_A$ :  $F_T(Y) < F_C(Y)$   
 SUM OF RANKS FOR FIRST TREATMENT = 155.00  
 CRITICAL VALUES ARE 0.0 AND 128.00  
 REJECT NULL HYPOTHESIS

C

$H_0$ : MEAN T  $\leq$  MEAN C  
 $H_A$ : MEAN T  $>$  MEAN C  
 CALCULATED  $T = 7.30$  CRITICAL  $T = 1.73$  D.F. = 18

AL  
PROPOSED SAMPLE 2

STATISTICS 510, T-TEST  
LAB.NO. 9

SEAT NO.  
CODE NO. 3

NOVEMBER 19, 1968

PROBLEM 1 10 POINTS

A CANDY COMPANY WAS STUDYING THE COST OF ITS PECAN PRALINES WITH THE INTENT OF REDUCING COST WITHOUT SACRIFICING FLAVOR. SINCE FLAVOR IS KNOWN TO BE CLOSELY ASSOCIATED WITH PERCENTAGE OF OILS IN NUT MEATS, FLAVOR CAN BE MAINTAINED BY REDUCING THE AMOUNT OF PECANS AS OIL CONTENT INCREASES. THIS PROMPTED THE COMPANY TO EXAMINE THE OIL CONTENT OF NORTHERN-GROWN (MISSOURI) AND SOUTHERN-GROWN (GEORGIA) PECANS BECAUSE THEY COULD BE PURCHASED FOR THE SAME PRICE. RANDOM SAMPLES OF PECANS YIELDED THE FOLLOWING:

OIL PERCENTAGES (BY WEIGHT)

NORTHERN	71.87	72.06	69.73	69.17	69.09	70.48
	69.59	70.39	71.80	70.60	70.94	
SOUTHERN	71.17	71.27	71.42	72.05	72.79	70.62
	71.08	72.34	71.77	71.21	73.20	73.50

A. THE COMPANY'S NORTHERN PURCHASED PECANS HAD A MEAN OIL PERCENTAGE OF 71 LAST YEAR. IS THE CURRENT DATA CONSISTENT WITH LAST YEAR'S DATA? (USE ONLY THE NORTHERN DATA AND LET  $\alpha = .01$ )

B. CONSTRUCT A .95 CONFIDENCE INTERVAL ON THE POPULATION MEAN OIL PERCENTAGE FOR SOUTHERN-GROWN PECANS.



C. PAST INFORMATION SUGGESTS THAT NORTHERN-GROWN PECANS SHOULD GAVE THE GREATEST AMOUNT OF OIL. DOES THE DATA SUBSTANTIATE THIS? (ASSUME EQUAL VARIANCES AND USE  $\alpha = .05$ )

D. CONSTRUCT A .95 CONFIDENCE INTERVAL ABOUT THE COMMON VARIANCE FOR THE POPULATION, ASSUMING OF COURSE THAT THEY ARE EQUAL. (USE ALL OF THE DATA.)

AL EX  
PROPOSED SAMPLE 2

STATISTICS 510, T-TEST  
LAB.NO. 9

SEAT NO.  
CODE NO. 3

NOVEMBER 19, 1968

PROBLEM 2 7 POINTS

A NUTRITION LABORATORY IS INTERESTED IN COMPARING TWO METHODS FOR MEASURING THE AMOUNT OF AMINO ACID SYNTHESIS UNDER VARIOUS DIETS. HOWEVER, THE MEASUREMENTS ARE TECHNICIAN DEPENDENT, I.E., SEVERAL TECHNICIANS WOULD PRODUCE DIFFERENT MEAN METHODS FOR THE SAME SAMPLE. SINCE AN APPROPRIATE EXPERIMENTAL DESIGN AND ANALYSIS COULD BE EMPLOYED TO REMOVE THE EFFECT DUE TO TECHNICIANS, THE NUTRITION LABORATORY DECIDED TO LET EACH OF THEIR 12 TECHNICIANS ANALYZE 10 AMINO ACID SAMPLES UNDER EACH METHOD OF MEASUREMENT. THE MEAN MEASUREMENTS OF THE TEN SAMPLES (IN MILLILITERS) FOR THE 11 TECHNICIAN ARE GIVEN BELOW.

	TECHNICIAN							
	1	2	3	4	5	6	7	8
METHOD 1	1.79	2.37	2.75	2.81	1.42	1.47	1.72	2.34
METHOD 2	1.63	2.10	2.78	2.61	0.75	0.73	1.74	2.04
	9	10	11					
METHOD 1	1.41	1.39	1.97					
METHOD 2	1.41	0.94	1.64					

- A. A MEAN OF TEN SAMPLES WAS USED AS A MEASUREMENT INSTEAD OF USING THE INDIVIDUAL OBSERVATIONS. WHAT EFFECT DOES THIS HAVE ON THE VARIANCE? WHAT DISTRIBUTION WOULD YOU EXPECT THE DATA TO FOLLOW? EXPLAIN.
- B. PLACE A .95 CONFIDENCE INTERVAL AROUND THE TRUE MEAN DIFFERENCE FOR THE TWO METHODS OF MEASUREMENT.
- C. CONSTRUCT A .95 CONFIDENCE INTERVAL FOR THE POPULATION VARIANCE OF THE DIFFERENCE BETWEEN THE TWO METHODS OF MEASUREMENT.

AL EX  
PROPOSED SAMPLE 2

STATISTICS 510, T-TEST  
LAB.NO. 9

SEAT NO.  
CODE NO. 3

NOVEMBER 19, 1968

PROBLEM 3 8 POINTS

TWENTY PLOTS OF GROUND WERE PLANTED TO CORN. A RANDOMLY SELECTED SET OF TEN PLOTS RECEIVED A NEW TYPE OF PHOSPHORUS FERTILIZER; THE REMAINDER RECEIVED NOTHING. DID THIS TREATMENT INCREASE THE YIELD?

TREATMENT 6.62 5.89 5.94 6.10 5.85 6.17 5.81 6.23 5.93 6.07

CONTROL 4.97 5.05 5.67 5.71 4.80 4.85 5.00 5.01 4.88 5.09

A. DESCRIBE THE POPULATION STRUCTURE AND FRAME THE EXPERIMENTAL QUESTION IN A STATISTICAL MANNER.

B. ANSWER THE EXPERIMENTAL QUESTION WITHOUT ASSUMING NORMALITY OF YIELD DISTRIBUTION.

C. ANSWER THE EXPERIMENTAL QUESTION ASSUMING NORMALITY OF YIELD DISTRIBUTION AND EQUALITY OF VARIANCES.

PROPOSED <sup>As Ex</sup> SAMPLE 2

(3)

## PROBLEM 1

10 POINTS

A

CALCULATED  $T = -1.48$  CRITICAL  $T = 3.17$  D.F. = 10  
 FAIL TO REJECT THAT MEAN = 71.0000

B

ESTIMATED MEAN = 71.87  
 CONFIDENCE INTERVAL IS  $P(71.28 \leq \text{MEAN} \leq 72.46) = .95$

C

CALCULATED  $T = -3.24$  CRITICAL  $T = 1.72$  D.F. = 21  
 FAIL TO REJECT THAT MEAN OF X = MEAN OF Y

D

ESTIMATED COMMON VARIANCE = 1.00  
 CONFIDENCE INTERVAL IS  $P(0.59 \leq \text{VARIANCE} \leq 2.03) = .95$

## PROBLEM 2

7 POINTS

A

NO NUMERICAL ANSWER  
 THE CENTRAL LIMIT THEOREM ASSURES APPROXIMATE NORMALITY. THE VARIANCE OF SUCH A MEAN IS LESS THAN THAT OF AN INDIVIDUAL OBSERVATION.

B

ESTIMATED MEAN = 0.28  
 CONFIDENCE INTERVAL IS  $P(0.10 \leq \text{MEAN} \leq 0.45) = .95$

C

ESTIMATED VARIANCE = 0.07  
 CONFIDENCE INTERVAL IS  $P(0.03 \leq \text{VARIANCE} \leq 0.21) = .95$

## PROBLEM 3

8 POINTS

A

NO NUMERICAL ANSWER  
 THERE IS A CONCEPTUAL POPULATION OF CORN YIELDS WHEN THE FERTILIZER IS APPLIED; ANOTHER WHEN IT IS NOT. WE HAVE UNRESTRICTED RANDOM SAMPLES FROM EACH OF THESE. ARE THEIR DISTR. THE SAME?

B

$H_0: F_T(Y) \geq F_C(Y)$   
 $H_A: F_T(Y) < F_C(Y)$   
 SUM OF RANKS FOR FIRST TREATMENT = 155.00  
 CRITICAL VALUES ARE 0.0 AND 128.00  
 REJECT NULL HYPOTHESIS

C

$H_0: \text{MEAN } T \leq \text{MEAN } C$   
 $H_A: \text{MEAN } T > \text{MEAN } C$   
 CALCULATED  $T = 7.52$  CRITICAL  $T = 1.73$  D.F. = 18  
 REJECT THAT MEAN OF X = MEAN OF Y IN FAVOR OF MEAN OF X > MEAN OF Y

ALEX  
PROPOSED SAMPLE 3

STATISTICS 510, T-TEST  
LAB.NO. 9

SEAT NO.  
CODE NO. 4

NOVEMBER 19, 1968

PROBLEM 1 10 POINTS

SOME LOCAL TEMPORARY PONDS HAVE TWO VERY CLOSELY RELATED SPECIES OF FAIRY SHRIMP COEXISTING IN THEM AT THE SAME TIME. ACCORDING TO THE COMPETITIVE EXCLUSION PRINCIPLE, THIS IS NOT POSSIBLE UNLESS THERE IS SOME SORT OF NICHE DIVERGENCE TO REDUCE INTERSPECIFIC COMPETITION. THESE CRUSTACEANS FILTER-FEED ON ALGAE, YEASTS, BACTERIA, DETRITUS, ETC. IF THEIR FILTERING APPENDAGES ARE DIFFERENT, THE DIFFERENCE MIGHT ALLOW NICHE DIVERGENCE BY SELECTING DIFFERENT SIZED FOOD PARTICLES. MATURE INDIVIDUALS WERE RANDOMLY DRAWN FROM PAST COLLECTION OF EACH SPECIES; THE 6TH THORACIC APPENDAGE WAS CLIPPED OFF AND MOUNTED ON A SLIDE. THESE APPENDAGES BEAR FILTERING SETAE (HAIR-LIKE SPINES); ONLY THE CENTRAL SETA WAS MEASURED VIA AN OCULAR MICROMETER. THESE MEASUREMENTS (IN MM/10) WERE

SPECIES A 0.21 0.17 0.15 0.15 0.18 0.17 0.20 0.20 0.13 0.16 0.16

SPECIES B 0.22 0.24 0.23 0.26 0.24 0.27 0.23 0.26 0.23 0.23 0.22 0.25

- A. IN ANOTHER POND ENVIRONMENT, THE MEAN SETAE LENGTH FOR SPECIES A WAS .17. IS THIS ABOVE DATA CONSISTENT WITH THIS? (USE ONLY SPECIES A DATA AND LET  $\alpha = .01$ )

- B. CONSTRUCT A .95 CONFIDENCE INTERVAL ON MEAN SETAE LENGTH FOR SPECIES B.

C. DO THE SPECIES DIFFER WITH REGARD TO MEAN SETAE LENGTH? (ASSUME EQUAL VARIANCES AND LET  $\alpha = .05$ )

D. CONSTRUCT A .95 CONFIDENCE INTERVAL ABOUT THE COMMON VARIANCE FOR THE TWO POPULATIONS, ASSUMING OF COURSE THAT THEY ARE EQUAL. (USE ALL OF THE DATA.)

AL EX  
PROPOSED SAMPLE 3

STATISTICS 510, T-TEST  
LAB.NO. 9

SEAT NO.  
CODE NO. 4

NOVEMBER 19, 1968

PROBLEM 2 7 POINTS

A NUTRITION LABORATORY IS INTERESTED IN COMPARING TWO METHODS FOR MEASURING THE AMOUNT OF AMINO ACID SYNTHESIS UNDER VARIOUS DIETS. HOWEVER, THE MEASUREMENTS ARE TECHNICIAN DEPENDENT, I.E., SEVERAL TECHNICIANS WOULD PRODUCE DIFFERENT MEAN METHODS FOR THE SAME SAMPLE. SINCE AN APPROPRIATE EXPERIMENTAL DESIGN AND ANALYSIS COULD BE EMPLOYED TO REMOVE THE EFFECT DUE TO TECHNICIANS, THE NUTRITION LABORATORY DECIDED TO LET EACH OF THEIR 12 TECHNICIANS ANALYZE 10 AMINO ACID SAMPLES UNDER EACH METHOD OF MEASUREMENT. THE MEAN MEASUREMENTS OF THE TEN SAMPLES (IN MILLILITERS) FOR THE 11 TECHNICIAN ARE GIVEN BELOW.

	TECHNICIAN							
	1	2	3	4	5	6	7	8
METHOD 1	1.46	1.75	1.52	1.86	1.21	0.97	1.18	1.84
METHOD 2	0.99	1.96	1.16	1.28	0.81	0.31	1.09	1.68
	9	10	11					
METHOD 1	1.08	2.16	1.62					
METHOD 2	0.80	1.54	1.41					

A. A MEAN OF TEN SAMPLES WAS USED AS A MEASUREMENT INSTEAD OF USING THE INDIVIDUAL OBSERVATIONS. WHAT EFFECT DOES THIS HAVE ON THE VARIANCE? WHAT DISTRIBUTION WOULD YOU EXPECT THE DATA TO FOLLOW? EXPLAIN.

B. PLACE A .95 CONFIDENCE INTERVAL AROUND THE TRUE MEAN DIFFERENCE FOR THE TWO METHODS OF MEASUREMENT.

C. CONSTRUCT A .95 CONFIDENCE INTERVAL FOR THE POPULATION VARIANCE OF THE DIFFERENCE BETWEEN THE TWO METHODS OF MEASUREMENT.

PROPOSED <sup>AL</sup>~~ED~~ <sup>EX</sup>SAMPLE 3

STATISTICS 510, T-TEST  
LAB.NO. 9

SEAT NO.  
CODE NO. 4

NOVEMBER 19, 1968

PROBLEM 3 8 POINTS

TWENTY PLOTS OF GROUND WERE PLANTED TO CORN. A RANDOMLY SELECTED SET OF TEN PLOTS RECEIVED A NEW TYPE OF PHOSPHORUS FERTILIZER; THE REMAINDER RECEIVED NOTHING. DID THIS TREATMENT INCREASE THE YIELD?

TREATMENT 5.77 6.07 6.03 5.69 5.73 5.89 6.72 5.37 6.93 6.84

CONTROL 4.78 4.86 5.32 4.89 5.00 4.85 4.77 5.11 4.74 5.06

- A. DESCRIBE THE POPULATION STRUCTURE AND FRAME THE EXPERIMENTAL QUESTION IN A STATISTICAL MANNER.
- B. ANSWER THE EXPERIMENTAL QUESTION WITHOUT ASSUMING NORMALITY OF YIELD DISTRIBUTION.
- C. ANSWER THE EXPERIMENTAL QUESTION ASSUMING NORMALITY OF YIELD DISTRIBUTION AND EQUALITY OF VARIANCES.



PROPOSED <sup>AL EX</sup> SAMPLE 3

(4)

## PROBLEM 1

10 POINTS

A

CALCULATED T = 0.12 CRITICAL T = 3.17 D.F. = 10  
 FAIL TO REJECT THAT MEAN = 0.1700

B

ESTIMATED MEAN = 0.24  
 CONFIDENCE INTERVAL IS  $P(0.23 \leq \text{MEAN} \leq 0.25) = .95$

C

CALCULATED T = -7.95 CRITICAL T = -2.08 D.F. = 21  
 REJECT THAT MEAN OF X = MEAN OF Y IN FAVOR OF MEAN OF X / = MEAN OF Y

D

ESTIMATED COMMON VARIANCE = 0.00  
 CONFIDENCE INTERVAL IS  $P(0.00 \leq \text{VARIANCE} \leq 0.00) = .95$

## PROBLEM 2

7 POINTS

A

NO NUMERICAL ANSWER  
 THE CENTRAL LIMIT THEOREM ASSURES APPROXIMATE NORMALITY. THE VARIANCE OF SUCH A MEAN IS LESS THAN THAT OF AN INDIVIDUAL OBSERVATION.

B

ESTIMATED MEAN = 0.33  
 CONFIDENCE INTERVAL IS  $P(0.15 \leq \text{MEAN} \leq 0.50) = .95$

C

ESTIMATED VARIANCE = 0.07  
 CONFIDENCE INTERVAL IS  $P(0.03 \leq \text{VARIANCE} \leq 0.21) = .95$

## PROBLEM 3

8 POINTS

A

NO NUMERICAL ANSWER  
 THERE IS A CONCEPTUAL POPULATION OF CORN YIELDS WHEN THE FERTILIZER IS APPLIED; ANOTHER WHEN IT IS NOT. WE HAVE UNRESTRICTED RANDOM SAMPLES FROM EACH OF THESE. ARE THEIR DISTR. THE SAME?

B

$H_0: F_T(Y) \geq F_C(Y)$   
 $H_A: F_T(Y) < F_C(Y)$   
 SUM OF RANKS FOR FIRST TREATMENT = 155.00  
 CRITICAL VALUES ARE 0.0 AND 128.00  
 REJECT NULL HYPOTHESIS

C

$H_0: \text{MEAN } T \leq \text{MEAN } C$   
 $H_A: \text{MEAN } T > \text{MEAN } C$   
 CALCULATED T = 6.47 CRITICAL T = 1.73 D.F. = 18  
 REJECT THAT MEAN OF X = MEAN OF Y IN FAVOR OF MEAN OF X > MEAN OF Y

SKELETON BUDGETS

Project 1: Problem Generation

Period: 7/1/69 - 8/31/70

Faculty - 1/5 time academic year	\$ 2,600
- 1/2 time 4 summer months	3,000
Graduate Ass'ts. (2) academic year	10,000
4 summer months	4,800
Supplies (cards and special forms)	200
Computing Expenses (1/2 hour per week x 60 weeks x \$225/hour)	6,750
	<hr/>
	\$27,350

Project 2: Problem Generation with Student-Computer Interaction

Period: 7/1/69 - 8/31/72

	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>
Faculty - 1/5 time academic year	\$ 2,600	\$ 2,700	\$ 2,800
- 1/2 time 2 summer months	1,500	1,600	1,700
Research Assoc. (1)	16,000	16,500	17,000
Systems Programmer	15,000 (1 $\frac{1}{4}$ )	18,000 (1 $\frac{1}{2}$ )	6,000 ( $\frac{1}{2}$ )
Graduate Ass'ts. - academic year	10,000	10,400	10,800
- 2 summer months	2,400	2,400	2,400
Secretarial and Publication	500	500	1,500
Travel	2,500	500	1,000
Supplies	200	200	300
Terminal Expenses (\$250/month)	1,000 (4 mos.)	3,000 (12 mos.)	5,000 (12 mos. + 2 x 4 mos.)
Computer Charges	7,500	11,000	13,000
Totals - - - - -	<hr/> \$59,200	<hr/> \$66,800	<hr/> \$61,500